Modeling of Hyper-Redundant Manipulators Dynamics and Design of Fuzzy Controller for the System

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Abstract—*Hyper-redundant manipulators having degrees of freedom much more than required, have many advantages and important capabilities. In this paper, modeling of the manipulator dynamics is done using a special curve called 'backbone' curve, modal method, Lagrangian mechanics and geometric transformation between variables space of backbone curve and manipulator joints. The dependency of the nonlinear and coupled terms of the dynamics model to joint variables makes some difficulties in classical methods to controller design. To overcome this problem, fuzzy controllers that have appropriate efficiency in complex and nonlinear systems are used. For demonstrating this matter, dynamics modeling of 10 degrees of freedom manipulator is done. Then a fuzzy controller is designed with attention to the dynamics behavior of the system. Manipulator behavior through various and noisy inputs are evaluated by simulation of the model including fuzzy controller. The results show very small error in manipulator motions and suitable condition of the designed fuzzy controller based on the dynamics model.*

1. INTRODUCTION

Hyper-redundant manipulators have a large or infinite degree of kinematics redundancy. They are analogous in morphology to snake, tentacles, and elephant trunks. Their high degree of articulation makes them work well suited in highly constrained environments, such as nuclear reactor cores, underground toxic waste tanks, or the human intestine.

Hyper-redundant manipulators can be implemented in a variety of physical morphologies. The possible morphologies can be roughly categorized into three main types (see figure 1). In figure 1.a, a continuous morphology is shown. Figure 1.b shows a variable geometry truss structure or VGT. Figure 1.c shows a discrete morphology i.e. one with a large, but finite, number of rigid links. References [1], [2], [3] and [4] should be consulted for details of different morphologies.

This paper is focused on inextensible planar hyper-redundant manipulators with discrete morphologies. It must be noticed that manipulators with fixed length are referred to as inextensible, whereas those which can change length are called extensible.

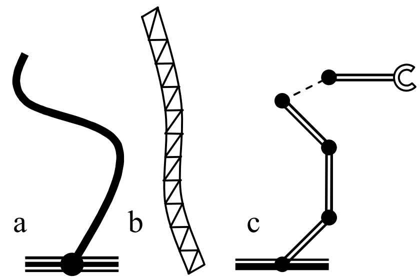

Figure 1 - different morphologies

To resolve the kinematic redundancy problem of hyper-redundant manipulators, different schemes were introduced, i.e., Pseudo inverse [6], generalized inverse [7], and extended inverse [8], of the manipulator Jacobian matrix. These schemes, however, are difficult to implement in the real time control when including the dynamic effect of the arm into consideration because of their computational inefficiency in manipulation of matrices and nonlinear terms. In this paper we use curvilinear or backbone curve theory [9], [10], [11].In this theory, it is assumed that regardless of mechanical implementation the important macroscopic features of a hyper-redundant manipulator can be captured by a backbone curve.

The dynamics of hyper-redundant manipulators was first formulated macroscopically by Chirikjian [12]. He used the principles of continuum mechanics to approximately represent the dynamics of hyper-redundant manipulators, where the dynamics of the continuum mechanics is first formulated and then projected onto the actual physical structure. This modeling technique, however, is only an approximation. S.Ma, Watanabe and Condo, formulated the dynamics of hyper-redundant manipulators accurately in a parameterized form and proposed a control dynamic scheme for hyper-redundant manipulators where the manipulator dynamics is included into consideration [13]. In this paper we use Lagrangian mechanics to derive the dynamics equations of hyper-redundant manipulators. The dependency of the nonlinear and coupled terms of the dynamics model to joint variables make some difficulties in classical methods to controller design [5], [19], [20]. To overcome this problem, fuzzy controllers that have appropriate efficiency in complex and nonlinear systems are used.

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The paper is organized as follow: section 2 reviews the kinematics problem of hyper-redundant manipulators. Section 3 represents the dynamics modeling of hyper-redundant manipulators. Section 4 denotes the fuzzy controller design for the system. The computer simulation is performed and its results are given in section 5. Section 6 gives conclusion of the paper.

2. KINEMATICS OF HYPER-REDUNDANT MANIPULATORS

As it was stated before, different schemes, that were introduced traditionally, are difficult to implement in the real time control when including the dynamics effect of the manipulator into consideration. In this section we explain Backbone curve approach for inverse kinematics problem of hyper-redundant manipulators. Backbone curve theory was used to represent the posture of kinematically hyper-redundant manipulators with an assumption that a hyper-redundant manipulator can be captured by a continuous curve regardless of mechanical implementation [9].

This paper is focused on a planar inextensible hyper-redundant manipulator with discrete morphology which works in a workspace containing gravity effects (figure 1.c). In this case, as shown in figure 2, for i-th link of the hyper-redundant manipulator, let m_i be the mass, l_i be the length and J_i be the inertia, also for i-th joint let θ_i be the joint angle, $\dot{\theta}_i$ be the angular velocity, $\ddot{\theta}_i$ be the angular acceleration and τ_i be the actuated momentum into the joint.

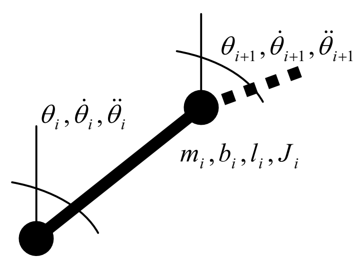

Figure 2 - details of i-th link

To resolve the kinematics problem of hyper-redundant manipulators using backbone curve theory, the arm posture of the hyper-redundant manipulator must be modeled by a continuous curve that called backbone curve. So the backbone curve that is shown in figure 3, can be defined as: Definition: A backbone curve is a piecewise continuous curve that captures the important macroscopic geometry features of a hyper-redundant manipulator (Figure 3).

The angle $\alpha(s,t)$ as shown in figure 3, represents the inclination angle of the vector with respect to x-axis on the curvilinear length. Where s is the distance along the curve measured from the base and t is time.

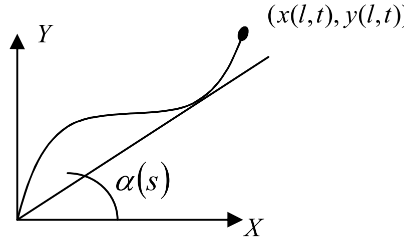

Figure 3 - Backbone curve

$(x(l,t), y(l,t))$ shows the coordinate of the end-effector. It must be understandable that the position of each point on the manipulator is time variable. In this manner, $\kappa(s,t)$ is the curvature function of the backbone curve and can be defined as:

$$\kappa(s,t) = \frac{2\pi}{l}\left(a_1(t)\cos(\frac{2\pi}{l}s) + a_2(t)\sin(\frac{2\pi}{l}s) \right) \tag{1}$$

Where a_1 and a_2 are the modal participation factors, [9] and l is the curve length which is equal to the length of the manipulator that is to be configured. The end-effector position can be derived through integration with respect to the curve length, and given by:

$$\begin{aligned}\alpha(s,t) &= \alpha_0 + \int_0^s \kappa(u,t)du = \\ &\alpha_0 + a_2(t) + a_1(t)\sin(\frac{2\pi}{l}s) + a_2(t)\cos(\frac{2\pi}{l}s)\end{aligned} \tag{2}$$

Where α_0 is the initial inclination angle of the vector with respect to x-axis at the start point or the arm base. The end-effector position elements $x(l,t), y(l,t)$ are derived by solving the equation (2), and given by (3) and (4).

$$\begin{aligned}x(l,t) &= \int_0^l \cos(\alpha(s,t))ds = \\ &\cos(\alpha_0 + a_2(t))J_0\left(\sqrt{a_1^2(t) + a_2^2(t)}\right)l\end{aligned} \tag{3}$$

$$\begin{aligned}y(l,t) &= \int_0^l \sin(\alpha(s,t))ds = \\ &\sin(\alpha_0 + a_2(t))J_0\left(\sqrt{a_1^2(t) + a_2^2(t)}\right)l\end{aligned} \tag{4}$$

Where J_0 is the zero order Bessel function. While the end position of the curve is given, its form given by the curvature $\kappa(s,t)$ corresponding to the given initial inclination angle α_0 can be defined by the coefficient a_1 and a_2. These coefficients are derived by solving the equations (3) and (4) and given by (5) and (6).

$$a_2(t) = \tan^{-1}\left(\frac{y(l)}{x(l)}\right) - \alpha_0 \tag{5}$$

$$a_1(t) = \sqrt{\left(\left[J_0^{-1}\left(\frac{\left(\sqrt{x(l)^2 + y(l)^2}\right)}{l}\right)\right]^2 - a_2^2\right)} \tag{6}$$

Where J_0^{-1} is the restricted inverse zero order Bessel function [13], [20]. The inverse solution of the curve is thus, derived and defined by the coefficients a_1 and a_2. Of course if we change the initial inclination angle α_0 the form of the curve would be changed too as shown in equation (2). Thus the form of the serpenoid curve is determined by three parameters a_1, a_2 and α_0 [13]. It should be noted here that, this technique highly restricts the working area and the flexibility of hyper-redundant manipulator by constraining the manipulator arm onto the serpenoid curve.

3. Dynamics of Hyper-Redundant Manipulators

In section 2, we used the backbone theory to solve forward and inverse kinematics problem of hyper-redundant manipulators. In this section, we first formulate the dynamics of hyper-redundant manipulators and then model the system dynamics.

As it denoted later, the arm posture of the hyper-redundant manipulator can be configured by restricting the arm on the defined serpenoid curve. In this case, the joint angles of the manipulator are derived from the grade of the tangent line, and can be expressed as:

$$\begin{aligned}
\theta_1(t) &= \alpha(\frac{L}{2}) = a_1(t)\sin(\frac{\pi}{n}) + a_2(t)\left(1-\cos(\frac{\pi}{n})\right) + \alpha_0 \\
\theta_i(t) &= \alpha\left((i-1+\frac{1}{2})L\right) - \alpha\left((i-1-\frac{1}{2})L\right) \\
&= a_1(t)\left[\sin\left(\frac{\pi}{n}(2i-1)\right) - \sin\left(\frac{\pi}{n}(2i-3)\right)\right] \\
&\quad - a_2(t)\left[\cos\left(\frac{\pi}{n}(2i-1)\right) - \cos\left(\frac{\pi}{n}(2i-3)\right)\right]
\end{aligned} \tag{7}$$

Where $i = 2,3,\ldots,n$ that n is number of links of the manipulator and L is the length of the link, equal to l/n. Rewriting equation (7) in vector form, we have:

$$\vec{\theta} = \tilde{J}_\lambda \vec{a} \tag{8}$$

In equation (8), $\tilde{J}_\lambda \in \Re^{n\times 3}$ is The Jacobian matrix, with elements given by $J_\lambda(i,j) = \theta_i / a_j$. In this case the elements of the Jacobian matrix are given by:

$$\begin{aligned}
\tilde{J}_\lambda(i,1) &= \begin{cases} \sin(\frac{\pi}{n}), & i=1 \\ \sin\left(\frac{\pi}{n}(2i-1)\right) - \sin\left(\frac{\pi}{n}(2i-3)\right), & i=2,3,\ldots,n \end{cases} \\
\tilde{J}_\lambda(i,2) &= \begin{cases} \left(1-\cos(\frac{\pi}{n})\right), & i=1 \\ \cos\left(\frac{\pi}{n}(2i-1)\right) - \cos\left(\frac{\pi}{n}(2i-3)\right), & i=2,3,\ldots,n \end{cases} \\
\tilde{J}_\lambda(i,3) &= \begin{cases} 1, & i=1 \\ 0, & i=2,3,\ldots,n \end{cases}
\end{aligned} \tag{9}$$

Differentiating equation (8) with respect to the time, we have:

$$\dot{\vec{\theta}} = \tilde{J}_\lambda \dot{\vec{a}} \tag{10}$$

As it shows, all of the elements of the Jacobian matrix are constant and time independent. So its time derivative $\dot{\tilde{J}}_\lambda \in \Re^{n\times 3}$ is thus the zero matrix. And the joint accelerations are become into:

$$\ddot{\vec{\theta}} = \tilde{J}_\lambda \ddot{\vec{a}} \tag{11}$$

It should be noted that this scheme makes the real time position control of hyper-redundant manipulators possible.
Same as joint angles, velocities and accelerations, the joint torques can also be represented in the parameterized form. The manipulator dynamics problem is generally formulated using techniques from Lagrangian mechanics or iterative Newton-Euler formulations. Lagrangian mechanics results in equations of motion of the form:

$$\vec{\tau} = \tilde{M}(\theta)\ddot{\vec{\theta}} + \tilde{C}(\theta,\dot{\theta}) + \vec{G}(\theta) \tag{12}$$

Where $\tilde{M} \in \Re^{n\times n}$ is the inertia matrix, $\tilde{C} \in \Re^{n\times n}$ is the torque matrix of Coriolis and centrifugal forces, $\vec{G} \in \Re^{n\times 1}$ is the torque vector of gravity force and $\vec{\tau} \in \Re^{n\times 1}$ is the torque vector actuated into joints, respectively. We can use equation (12) for any inextensible hyper-redundant manipulator with discrete morphology.

For a manipulator with n links, the equations of motion are $n\times n$ matrices, however, using modal approach the equations of motion would be reduced to assumed mode numbers (two harmonic modes, here). Now with applying equations (9), (10), (11), into equation (12), we have:

$$\tilde{M}(a)\left(\tilde{J}_\lambda \ddot{\vec{a}}\right) + \tilde{C}(a,\dot{a}) + \vec{G}(a) = \vec{\tau} \tag{13}$$

The dynamics equations of the hyper-redundant manipulator are derived in the joint variables space and transformed to the backbone curve variables space. Multiplying equation (13) by $\tilde{J}_\lambda^T$ we have:

$$\left(\tilde{J}_\lambda^T \tilde{M}(a)\tilde{J}_\lambda\right)\ddot{\vec{a}} + \tilde{J}_\lambda^T \tilde{C}(a,\dot{a}) + \tilde{J}_\lambda^T \vec{G}(a) = \tilde{J}_\lambda^T \vec{\tau} \tag{14}$$

Rewriting equation (14) in short form, we have:

$$\tilde{M}^*(a)\ddot{\vec{a}} + \tilde{C}^*(a,\dot{a}) + \vec{G}^*(a) = \vec{\tau}^* \tag{15}$$

Where we assume, $\tilde{M}^*(a)$ be $\tilde{J}_\lambda^T \tilde{M}(a)\tilde{J}_\lambda$, $\tilde{C}^*(a,\dot{a})$ be $\tilde{J}_\lambda^T \tilde{C}(a,\dot{a})$, $\vec{G}^*(a)$ be $\tilde{J}_\lambda^T \vec{G}(a)$ and $\vec{\tau}^*$ be $\tilde{J}_\lambda^T \vec{\tau}$ respectively. The deformed inertia matrix can be written as:

$$\tilde{M}^* = \tilde{M}_0^* + \tilde{M}_1^* \tag{16}$$

Where $\tilde{M}_0^*$ contains diagonal elements of the main inertia matrix, which these elements are constant values and $\tilde{M}_1^*$ contains all non-diagonal elements. Substituting equation (16) for equation (15), we have:

$$\tilde{M}_0^* \ddot{\vec{a}} = \vec{\tau}^* - \left[\tilde{M}_1^*(a)\ddot{\vec{a}} + \tilde{C}^*(a,\dot{a}) + \vec{G}^*(a)\right] \tag{17}$$

Rewriting equation (17) in form of a control law, we have:

$$\ddot{\vec{a}} = \tilde{M}_0^{*-1}\left(\vec{\tau}^* - \left[\tilde{M}_1^*(a)\ddot{\vec{a}} + \tilde{C}^*(a,\dot{a}) + \vec{G}^*(a)\right]\right) \tag{18}$$

In this section, we deformed the dynamics equations of hyper-redundant manipulators to organize a suitable control law. Finally, using equation (18), the model of the system dynamics, as shown in figure 4, can be configured.

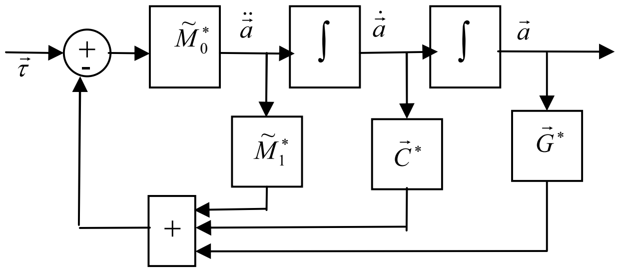

Figure 4 - Block diagram of the system dynamics

4. FUZZY CONTROL

In previous section, we formulated and modeled the dynamics of hyper-redundant manipulators using Lagrangian mechanics and modal approach including two harmonic modes. In this section we introduce a fuzzy controller for the hyper-redundant manipulator to track a desired path.

Basically, a fuzzy system consists of three main parts: the fuzzifire, the fuzzy inference engine and the defuzzifire. The fuzzifire maps a crisp input into some fuzzy sets. The fuzzy inference engine uses fuzzy IF-THEN rules from a rule base to reason for the fuzzy output. The output in fuzzy terms is converted back to crisp value by the defuzzifire. In this paper, we used Sugeno fuzzy rules [18] to synthesize out fuzzy logic controller, which adopts the following general fuzzy IF-THEN rule:

$$\begin{aligned} &IF\ Input1\ is\ X\ and\ Input2\ is\ Y\ then\ Output\ is \\ &Z = aX + bY + c \end{aligned} \tag{19}$$

We use the Sugeno inference method to design the fuzzy controller, because it is a universal approximation [15]. The difference between Sugeno and Mamdani methods is the shape of output functions, however the output functions for Sugeno method, are linear or constant. For a zero order Sugeno model, the output is a constant value (a=b=0).
In this paper we consider that the Sugeno inference, consists of product inference engine [15-18], singleton fuzzifire [15-18], and center average defuzzifire [15-18].

To design the fuzzy logic controller for the modeled system, we first, define the input variables:

$$\begin{aligned} \Delta x_1 &= \Delta a_1 \\ \Delta x_2 &= \Delta a_2 \\ x_3 &= \dot{x}_1 = \dot{a}_1 \\ x_4 &= \dot{x}_2 = \dot{a}_2 \end{aligned} \tag{20}$$

These four state variables are used to design the fuzzy controller and the outputs of the fuzzy controller are the joint torque values. Then we define a suitable range for each input and a constant value for each output with attention to the dynamics behavior of the system.

According to the dynamics behaviors of the system, we define a suitable range for each input and a constant value for each output. Now we must define membership functions. The used membership function can be one of Gaussian, triangular, or any other type of membership functions [17], [18]. In this paper, we use Bell membership function and assume its parameters are fixed during the process. Figure 5, shows a generalized Bell membership function.

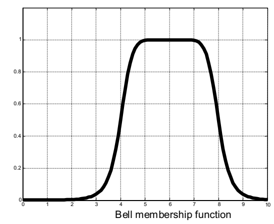

Figure 5 - Generalized Bell membership function

In the next step, we must organize a fuzzy rule base for the fuzzy controller. Totally, let w_i be output value of each step and z_i be the weight of each rule, it can be shown that:

$$Final\ Output = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i} \tag{21}$$

The final control system of the hyper-redundant manipulator can be shown in figure 6. It must be noted that any signal line contains a vector signal.

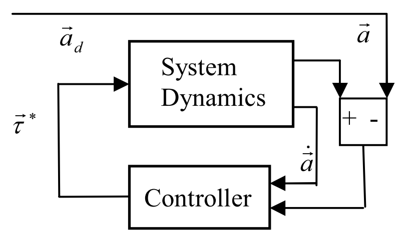

Figure 6 - Block diagram of control system

5. COMPUTER SIMULATION

We assume a 10-DOF (degree of freedom) hyper-redundant manipulator to evaluate the validity of the proposed method. The length of each link of the arm is 0.1 [m], the mass of each link is set as m=0.1 [kg] and inertia parameter is derived by seeing the link as a uniform beam. It should be noted here that the modal approach method is applied using two harmonic modes. The simulated dynamics model of the hyper-redundant manipulator is shown in figure 7.

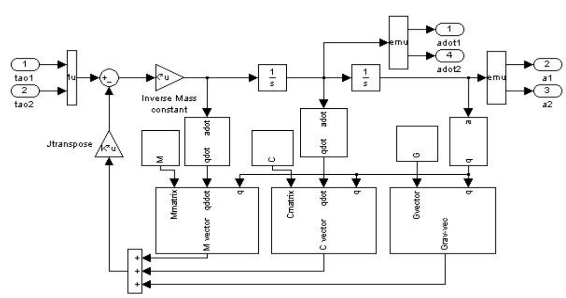

Figure 7 - Simulated dynamics model of the manipulator

To design a fuzzy controller for this model as described in section 4, first we assume four state variables and define suitable range for each one with attention to dynamics behavior of the system.

$$\begin{aligned} &\Delta x_1 \in [-0.3, 0.3]\,(Rad) \\ &\Delta x_2 \in [-0.3, 0.3]\,(Rad) \\ &x_3 \in [-1, 1]\,(Rad/Sec) \\ &x_4 \in [-1, 1]\,(Rad/Sec) \end{aligned} \tag{22}$$

Then we define constant values for the controller outputs.

$$\left.\begin{aligned} Very\ Negative\,(VN) &= -100\ (N.m) \\ Negative\,(N) &= -80\ (N.m) \\ Posetive\,(P) &= 80\ (N.m) \\ Very\ Posetive\,(VP) &= 100\ (N.m) \end{aligned}\right\} for\ (\tau_1) \tag{23}$$

And,

$$\left.\begin{aligned} Very\ Negative\,(VN) &= -50\ (N.m) \\ Negative\,(N) &= -40\ (N.m) \\ Posetive\,(P) &= 40\ (N.m) \\ Very\ Posetive\,(VP) &= 50\ (N.m) \end{aligned}\right\} for\ (\tau_2) \tag{24}$$

In this case we use two Bell membership function in each defined range (one in negative and other in positive region) of input variables.

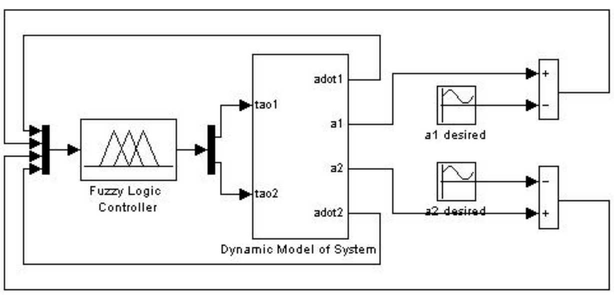

Figure 8 - Control System

To organize a fuzzy rule base for the fuzzy controller of the system, according to the equation (19), 16 distinct "IF-THEN" rules are defined and given in table 1:

Table 1 - IF-THEN Rules

Δx_1	N	N	N	N	N	N	N	N	P	P	P	P	P	P	P	P
Δx_2	N	N	P	P	N	N	P	P	N	N	P	P	N	N	P	P
x_3	N	N	N	N	P	P	P	P	N	N	N	N	P	P	P	P
x_4	N	P	N	P	N	P	N	P	N	P	N	P	N	P	N	P
τ_1	VP	VP	VP	VP	P	P	P	P	N	N	N	N	VN	VN	VN	VN
τ_2	VP	P	N	VN	VP	P	N	VN	VP	P	N	VN	VP	P	N	VN

The obtained fuzzy rule base is complete, continuous and consistent [18]. In this manner, the fuzzy inference engine is produced according to the equation (21). Then the resulted control system with four inputs and two outputs is placed in the fuzzy logic controller block of the simulated system, as shown in figure 8. Thereafter the simulated system is evaluated through various and noisy inputs. Totally, results show suitable condition of the designed fuzzy controller. Figure 9 shows number of obtained results. To make it clear we define a desired path in Cartesian space given by:

$$y = 0.4\sin(5x) + 0.2 \tag{25}$$

Where $0.08 \le x \le 0.4$. The goal of this experiment is to analyze the manipulator tracking ability and measure tracking error values as shown in figures 10 and 11. It must be noted that, paths consist of straight lines can be tracked exactly by the manipulator.

6. CONCLUSION

In this paper, we used backbone curve theory, modal method and Lagrangian mechanics to modeling of the manipulator dynamics. This technique is possible to implement to real-time control. For the hyper-redundant manipulators which are high order multivariable nonlinear systems and have coupled states, the fuzzy controllers indicate some advantages when compared to the other classical controllers. In computer simulation section, dynamics modeling of 10 DOF manipulator was done. Then a fuzzy controller was designed with attention to the dynamics behavior of the system. Thereafter the simulated system was evaluated through various and noisy inputs. Totally, results showed suitable condition and good performance of the designed fuzzy controller which made the hyper-redundant manipulator track the desired path accurately.

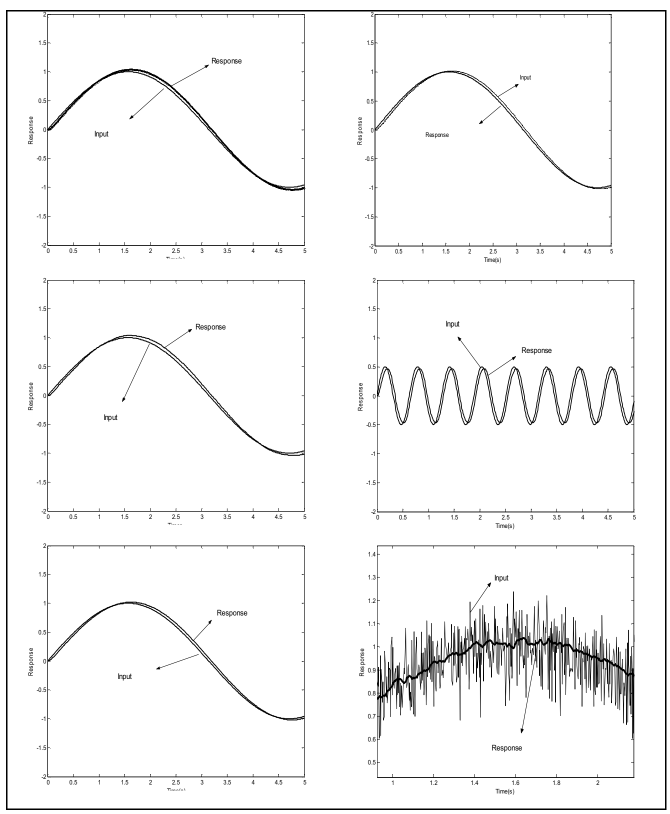

Figure 9 - Control System Response to different inputs

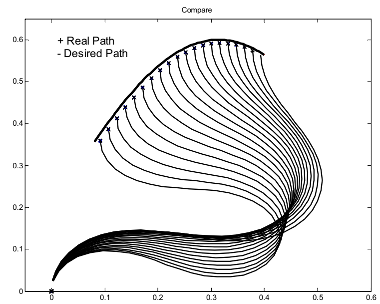

Figure 10 - Backbone curve configuration on tracking path

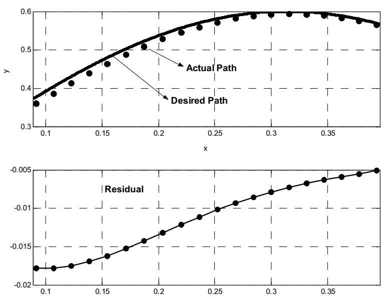

Figure 11 - Residual on tracking path

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